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Abstract
In this paper, we propose a quantum field theoretical renormalization group approach to the vortex dynamics of magnetically coupled layered superconductors, to supplement our earlier investigations on the Josephson-coupled case. We construct a two-dimensional multi-layer sine–Gordon type model which we map onto a gas of topological excitations. With a special choice of the mass matrix for our field theoretical model, vortex dominated properties of magnetically coupled layered superconductors can be described. The well known interaction potentials of fractional flux vortices are consistently obtained from our field theoretical analysis, and the physical parameters (vortex fugacity and temperature parameter) are also identified. We analyse the phase structure of the multi-layer sine–Gordon model by a differential renormalization group method for the magnetically coupled case from first principles. The dependence of the transition temperature on the number of layers is found to be in agreement with known results based on other methods.

1. Introduction

Recently, we have shown that layered sine–Gordon type models are probably not suitable for the description of Josephson-coupled layered superconductors, because the linear, confining potential that binds the vortices together cannot be obtained from the interaction of the topological excitations of the model, no matter how the interlayer interaction term is chosen [1]. On the other hand, vortex dominated properties of high T_c layered superconductors and other types of layered materials, e.g. superconducting sandwiches, have already received a considerable amount of attention (see, e.g., [2–16]), and the intuitively obvious connection of sine–Gordon models to these materials makes one wonder if at least a restricted applicability of

the layered, field theoretical model persists. We also observe that recently, there is increasing interest in the literature [17–19] in constructing sine–Gordon type field theoretical models in order to understand better the vortex dynamics in layered superconducting systems. Our aim in this paper to follow this route by constructing a two-dimensional multi-layer sine–Gordon type model which can be used to describe the vortex behaviour of magnetically as opposed to Josephson-coupled layered superconductors, and to contrast and enhance our recent investigation [1].

In a two-dimensional (2D) isolated superconducting thin film, the Pearl vortices [2, 14] are naturally identified as the topological excitations and can be considered as the charged analogues of the vortices in a 2D superfluid which generate the Kosterlitz–Thouless–Berezinski (KTB) phase transition [20]. The logarithmic interaction between the vortices of the superfluid extends to infinity and as a consequence they remain bound below the finite KTB transition temperature ($T_{\mathrm{KTB}}^{\star}$) and dissociate above it [20]. Since the Pearl vortices carry electric charge, they always remain unbound due to the screening length λ_{eff} generated by the electromagnetic field which cuts off the logarithmic interaction [4, 21, 22] and leads to the absence of any KTB phase transition. However, for realistic finite 2D superconducting films where the lateral dimension of the film can be smaller then the screening length $R_0 < \lambda_{\mathrm{eff}}$ the KTB transition can be restored [4, 21]. This constitutes an intrinsic finite size effect.

In layered materials, the interlayer coupling modifies the 2D picture and leads to new types of topological defects. If the layers are coupled by Josephson coupling (like for many HTSC materials) the vortex–antivortex pairs on the same layer interact with each other via a logarithmic term for small distances but they feel a linear confining potential for large distances (see e.g. [4] and references therein). The vortices in neighbouring layers always interact via a linear potential which can couple them by forming vortex loops, rings, or vortex 'strings' piercing all layers.

If the layers are coupled by purely magnetic interaction (e.g. in artificially produced superlattices where the Cooper pair tunnelling between the superconducting layers is suppressed by relatively large insulating layers) the topological defects for a system which consists of infinitely many layers are pancake vortices [10, 15] which undergo a KTB phase transition at $T_{\mathrm{KTB}}^{\star}$. As explained e.g. in [5], the Josephson coupling can be essentially neglected when the confinement length, i.e. the length scale at which the linear confining potential due to the Josephson coupling dominates over the logarithmic interaction due to magnetic effects, is pushed beyond the effective screening length for the logarithmic interaction among vortices. This situation is present when the tunnelling between the superconducting layers is suppressed by relatively large insulating layers, and a proposal for an experimental realization has recently been given [5]. For a finite number N of magnetically coupled layers, the Pearl type vortex stack [2] is broken up into a number of coupled pancake vortices of fractional flux [3, 13, 15], and this configuration undergoes a KTB type phase transition at a layer-dependent temperature $T_{\mathrm{KTB}}^{(N)} = T_{\mathrm{KTB}}^{\star}(1 - N^{-1})$ which is connected with the dissociation of the stack. This result has been obtained on the basis of the entropy method first introduced in the ground-breaking work [3]. Recently, a real space renormalization group (RG) analysis of the case $N = 2$ has been performed in [5] using the dilute gas approximation. *A priori*, it appears to be rather difficult to generalize this RG analysis for $N > 2$ layers.

In general, the Ginzburg–Landau (GL) theory [23] provides us with a good theoretical framework in which to investigate the vortex dynamics in thin films and in layered materials. Several equivalent models, like field-theoretical, statistical spin models and a gas of topological defects, have also been used to consider the vortex properties of films and layered systems. The 2D GL, 2D *XY* and the 2D Coulomb gas models (see e.g. [1, 4] and references therein) are considered as the appropriate theoretical background for the vortex dynamics of superfluid

films. The field theoretical counterpart is the 2D sine–Gordon (SG) model [24]. The two kinds of models belong to the same universality class and produce the KTB phase transition. For superconducting films one has to consider the 2D GL model in the presence of electromagnetic interactions [4] or the equivalent gas of topological excitations, the 2D Yukawa gas [21]. The corresponding field theory is the 2D SG model with an explicit mass term, the massive 2D SG model [21].

For Josephson-coupled layered superconductors in the case of very large anisotropy one should investigate the layered GL model including the Josephson coupling between the layers [4] (i.e. the Lawrence–Doniach model [25]). In the case of not too large anisotropy one can use the anisotropic, continuous GL theory [4, 23, 26] which can be mapped onto the isotropic GL model by an appropriate rescaling method [27]. The corresponding spin model is the 3D XY model [28] and the equivalent gases of topological excitations are the layered vortex [29] or vortex loop [28] gases. There are attempts in the literature to construct the field theoretical counterpart of the isotropic model [30]. In the case of strong anisotropy, the layered sine–Gordon (SG) model [31] has been proposed as a candidate model where the interlayer interaction between the topological defects has been described by a mass matrix which couples the SG fields

$$\frac{1}{2}\underline{\varphi}^{\mathrm{T}}\underline{\underline{m}}^2\underline{\varphi} \equiv \sum_{n=1}^{N-1}\frac{J}{2}(\varphi_{n+1}-\varphi_n)^2,$$

where $\underline{\varphi} = (\varphi_1,\ldots,\varphi_N)$ and φ_n $(n = 1,\ldots,N)$ are one-component scalar fields. Recently, we showed in [1] that the layered SG model with the above mass matrix is not appropriate for the description of vortex dynamics of Josephson-coupled layered superconductors.

In the case of purely magnetically coupled layered systems, the layered GL model has to be used, but excluding the Josephson coupling. Although the interaction potentials between the topological defects of magnetically coupled layered systems are given in [5, 8, 12, 15], no field theoretical model has been proposed for the description of vortex dynamics in a finite system of magnetically coupled superconductors.

Here, our aim is to open a new platform for considering the vortex dynamics of magnetically coupled layered systems by constructing a multi-layer sine–Gordon (MLSG) type field theoretical model where the two-dimensional sine–Gordon (2D SG) fields characterizing the layers are coupled by an appropriate general mass matrix,

$$\tfrac{1}{2}\underline{\varphi}^{\mathrm{T}}\underline{\underline{M}}^2\underline{\varphi} \equiv \tfrac{1}{2}G\left(\sum_{n=1}^{N}\varphi_n\right)^2.$$

By the exact mapping of the MLSG model onto an equivalent gas of topological defects, we recover the interaction potential given in [5, 8, 12, 15] and, hence, prove the applicability of the model. We analyse the phase structure of the MLSG model by a differential renormalization group (RG) method performed in momentum space, which is in general easier to perform than that in real space, and determine the layer dependence of $T_{\mathrm{KTB}}^{(N)}$. In our field theoretical RG approach, the RG flow can be calculated in one step for an arbitrary number of layers, and the study of the intrinsic finite size effect of thin film superconductors [4, 21] and of finite layered systems is facilitated.

This paper is organized as follows. In section 2, we define the multi-layer sine–Gordon model and show by its exact mapping onto the equivalent gas of topological excitations that it is suitable for describing the vortex dominated properties of magnetically coupled layered superconductors. In section 3, a renormalization group analysis of the multi-layer sine–Gordon model is performed within the framework of the Wegner—Houghton renormalization group method, in momentum space for general N, and with a solution that spans the entire domain

from the ultraviolet (UV) to the infrared (IR). The layer number dependence of the critical temperature of the multi-layer sine–Gordon model is determined by using the mass-corrected linearized RG flow. Conclusions are reserved for section 4.

2. The multi-layer sine–Gordon model

The multi-layer sine–Gordon (MLSG) model consists of N coupled two-dimensional sine–Gordon (2D SG) models of identical 'frequency' b, each of which corresponds to a single layer described with the scalar fields $\varphi_n(n = 1, 2, \ldots, N)$. Its Euclidean bare action (we imply here the sum over $\mu = 1, 2$)

$$S[\underline{\varphi}] = \int d^2 r[\tfrac{1}{2}(\partial_\mu \underline{\varphi})^{\mathrm{T}}(\partial_\mu \underline{\varphi}) + V(\underline{\varphi})] \tag{1}$$

contains the interaction terms

$$V(\underline{\varphi}) = \tfrac{1}{2}\underline{\varphi}^{\mathrm{T}}\underline{\underline{M}}^2\underline{\varphi} - \sum_{n=1}^{N} y_n \cos(b\varphi_n) \tag{2}$$

with the O(N) multiplet $\underline{\varphi} = (\varphi_1, \ldots, \varphi_N)$. We can choose the fugacities $y_n > 0$ without loss of generality, ensuring that the zero-field configuration is a local minimum of the action (see chapter 31 of [32]). The mass matrix describes the interaction between the layers and is chosen here to be of the form

$$\underline{\varphi}^{\mathrm{T}}\underline{\underline{M}}^2\underline{\varphi} = G\left(\sum_{n=1}^{N} a_n\varphi_n\right)^2, \tag{3}$$

where G is the strength of the interlayer interactions, and the a_n are free parameters. As will be explained below, any choice with $a_n^2 = 1$ for all $n = 1, \ldots, N$ reproduces exactly the same layer dependence of $T_{\mathrm{KTB}}^{(N)}$ as found in [3, 5]. In this case, the layers can be assumed to be equivalent and, as a consequence, the fugacity $y_n \equiv y$ for $n = 1, 2, \ldots, N$. The most obvious choice fulfilling $a_n^2 = 1$, namely $a_n = 1$ for all $n = 1, \ldots, N$, reproduces the interlayer interaction between pancake vortices given, e.g., in equation (89) of [15], and we will restrict our attention to this choice in the following.

The MLSG model has a discrete symmetry under the shift of the field variable $\underline{\varphi} \to \underline{\varphi} + \underline{\Delta}$ with $\underline{\Delta} = (l_1 2\pi/b, \ldots, l_N 2\pi/b)$ where the 'last' integer $l_N = -\sum_{n=1}^{N-1} l_n$ is fixed but all the other integers l_n ($n = 1, \ldots, N-1$) can be chosen freely (to see this, one just diagonalizes the mass matrix). The single non-vanishing mass eigenvalue is $M_N = \sqrt{NG}$, and hence the model possesses $N - 1$ massless 2D SG fields and a single massive 2D SG field. After the diagonalization of the mass matrix by a suitable rotation of the fields, the model thus is invariant under the independent separate shifts of $N - 1$ massless fields, but the explicit mass term of the single massive mode breaks the periodicity in the 'massive' direction of the N-dimensional internal space.

One crucial observation is that the partition function of the MLSG model, whose path integral formulation reads

$$\mathcal{Z} = \mathcal{N} \int [\mathcal{D}\underline{\varphi}] \exp(-S[\underline{\varphi}]), \tag{4}$$

can be identically rewritten in terms of an equivalent gas of topological excitations (vortices), whose interaction potentials are exactly equivalent to those of [5, 8, 12]. This finding constitutes a generalization of known connections of the d-dimensional globally neutral Coulomb gas and the d-dimensional sine–Gordon model, as discussed in chapter 32 of [32], and can be seen

as follows. In equation (1), one artificially introduces the vectors $f_n \equiv (\delta_{1n}, \ldots, \delta_{Nn})$ as projection operators to rewrite $\sum_{n=1}^{N} \cos(b\varphi_n) = \sum_{n=1}^{N} \cos(b \underline{f}_n^{\mathrm{T}} \underline{\varphi})$, one expands the periodic piece of the partition function (4) in a Taylor series, and one introduces the integer-valued charges $\sigma_\alpha = \pm 1$ of the topological defects which are subject to the neutrality condition $\sum_{\alpha=1}^{2\nu} \sigma_\alpha = 0$. This leads to the intermediate result

$$
\mathcal{Z} = \mathcal{N} \sum_{\nu=0}^{\infty} \frac{(y/2)^{2\nu}}{(2\nu)!} \prod_{i=1}^{2\nu} \left(\sum_{n_i=1}^{N} \int \mathrm{d}^2 r_i \right) \sum_{\substack{\sigma_1,\ldots,\sigma_\nu=\pm 1 \\ \sigma_{\nu+\gamma}=-\sigma_\gamma, \gamma \in \{1,\ldots\nu\}}}
$$
$$
\times \int \mathcal{D}[\underline{\varphi}] \exp\left[-\int \mathrm{d}^2 r \, \tfrac{1}{2} \underline{\varphi}^{\mathrm{T}} (-\partial^2 + \underline{\underline{M}}^2) \underline{\varphi} + \mathrm{i}b\underline{\rho}^{\mathrm{T}} \underline{\varphi} \right], \tag{5}
$$

where $\partial^2 \equiv \partial_\mu \partial_\mu$, and

$$
\underline{\rho}(r) = \sum_{\alpha=1}^{2\nu} \sigma_\alpha \delta(r - r_\alpha) \underline{f}_{n_\alpha}. \tag{6}
$$

We have thus placed the 2ν vortices, labelled with the index i, onto the N layers, with vortex i being placed onto the layer n_i. The Gaussian integration in equation (5) can now be performed easily, and the inversion of the matrix $-\partial^2 + \underline{\underline{M}}^2$ can be accomplished by going to momentum space. Via a subsequent back-transformation to coordinate space, we finally arrive at the result

$$
\mathcal{Z} = \sum_{\nu=0}^{\infty} \frac{(y/2)^{2\nu}}{(2\nu)!} \left(\prod_{i=1}^{2\nu} \sum_{n_i=1}^{N} \int \mathrm{d}^2 r_i \right) \sum_{\substack{\sigma_1,\ldots,\sigma_\nu=\pm 1 \\ \sigma_{\nu+\gamma}=-\sigma_\gamma, \gamma \in \{1,\ldots\nu\}}}
$$
$$
\times \exp\left[-\frac{b^2}{2} \sum_{\alpha,\gamma=1}^{2\nu} \sigma_\alpha \sigma_\gamma (\delta_{n_\alpha n_\gamma} A_{\alpha\gamma} + (1 - \delta_{n_\alpha n_\gamma}) B_{\alpha\gamma}) \right], \tag{7}
$$

where δ_{nm} represents the Kronecker delta. Equation (7) implies that the parameter b^2 in equation (2) can naturally be identified as being proportional to the inverse of the temperature of the gas, $b^2 \propto T^{-1}$. The potentials $A_{\alpha\gamma} \equiv A(\vec{r}_\alpha, \vec{r}_\gamma)$ and $B_{\alpha\gamma} \equiv B(\vec{r}_\alpha, \vec{r}_\gamma)$ are the intralayer and interlayer interaction potentials, respectively. They read

$$
A_{\alpha\gamma} = -\frac{1}{2\pi} \frac{N-1}{N} \ln\left(\frac{r_{\alpha\gamma}}{a} \right) + \frac{1}{2\pi} \frac{1}{N} \left[K_0\left(\frac{r_{\alpha\gamma}}{\lambda_{\mathrm{eff}}} \right) - K_0\left(\frac{a}{\lambda_{\mathrm{eff}}} \right) \right]
$$
$$
= \begin{cases} -\dfrac{1}{2\pi} \ln\left(\dfrac{r_{\alpha\gamma}}{a} \right) & (r_{\alpha\gamma} \ll \lambda_{\mathrm{eff}}) \\[12pt] -\dfrac{1}{2\pi} \left[\dfrac{N-1}{N} \ln\left(\dfrac{r_{\alpha\gamma}}{\lambda_{\mathrm{eff}}} \right) - \ln\left(\dfrac{\lambda_{\mathrm{eff}}}{a} \right) \right] & (r_{\alpha\gamma} \gg \lambda_{\mathrm{eff}}), \end{cases} \tag{8a}
$$

where $r_{\alpha\gamma} = |\vec{r}_\alpha - \vec{r}_\gamma|$, and

$$
B_{\alpha\gamma} = \frac{1}{2\pi} \frac{1}{N} \left(\ln\left(\frac{r_{\alpha\gamma}}{a} \right) + \left[K_0\left(\frac{r_{\alpha\gamma}}{\lambda_{\mathrm{eff}}} \right) - K_0\left(\frac{a}{\lambda_{\mathrm{eff}}} \right) \right] \right)
$$
$$
= \begin{cases} 0 & (r_{\alpha\gamma} \ll \lambda_{\mathrm{eff}}) \\[12pt] \dfrac{1}{2\pi} \dfrac{1}{N} \ln\left(\dfrac{r_{\alpha\gamma}}{\lambda_{\mathrm{eff}}} \right) & (r_{\alpha\gamma} \gg \lambda_{\mathrm{eff}}). \end{cases} \tag{8b}
$$

$K_0(r)$ stands for the modified Bessel function of the second kind, a is the lattice spacing which serves as an UV cut-off and an effective screening length λ_{eff} is introduced which is related inversely to the non-zero mass eigenvalue of the mass matrix (3), $\lambda_{\text{eff}}^{-1} = M_N = \sqrt{NG}$. The relation $K_0(r) = -\ln(r) + \ln 2 - \gamma_{\text{E}} + \mathcal{O}(r)$ has been used in the derivation of the asymptotic short- and long-range forms in equations (8a) and (8b), and only the leading logarithmic terms are indicated ($\gamma_{\text{E}} = 0.577\,216\ldots$ is Euler's constant).

The interaction potentials (8) have the same asymptotic behaviour as the vortices of magnetically coupled superconducting layers [5, 8, 12, 15] (for the intralayer and interlayer interactions see equations (86) and (89) of [15], under the substitution $\Lambda_D = \Lambda_s/N$). This observation shows that the MLSG field theory is suitable for describing the vortex dynamics in magnetically coupled layered systems. A few remarks are now in order. (i) The prefactor $(N - 1)/N$ appearing in the intralayer interaction indicates the existence of vortices with fractional flux in the MLSG model. (ii) For small distances $r \ll \lambda_{\text{eff}}$, the interlayer interaction B disappears and the intralayer potential A has the same logarithmic behaviour with full flux as that of the pure 2D SG model (which belongs to the same universality class as the 2D XY model and the 2D Coulomb gas). Therefore, the MLSG model for small distances behaves as an uncoupled system of 2D SG models. (iii) For the case $N = 1$, there exists no interlayer interaction, and the intralayer potential is logarithmic for small distances and vanishes for large distances. Consequently, there are always free, non-interacting vortices in the model which push the KTB transition temperature to zero. The MLSG model for a single layer reduces to the massive 2D SG model discussed in [4, 18, 21, 22] where the periodicity in the internal space is broken and the KTB transition is absent. (iv) In the bulk limit $N \to \infty$, the effective screening length and the interlayer interaction disappear ($\lambda_{\text{eff}} \to 0$, $B_{\alpha\gamma} \to 0$), and the intralayer potential has a logarithmic behaviour with full flux; thus the MLSG model predicts the same behaviour as that of the pure 2D SG model with $T_{\text{KTB}}^{(\infty)} = T_{\text{KTB}}^{\star}$. Alternatively, one may observe that for $N \to \infty$, the effect of the infinitely many zero-mass modes dominates over the effect of the single remaining massive mode entirely, leading to a constant limit for the transition temperature as $N \to \infty$.

For $N = 2$ layers, the MLSG model (with the choice $a_n = (-1)^{n+1}$) has been proposed for describing the vortex properties of Josephson-coupled layered superconductors [31]. However, the above discussed mapping indicates that any layered sine–Gordon model, whatever the mass matrix, can be mapped onto an equivalent gas of topological excitations, whose interaction potentials are determined by the inversion of a two-dimensional propagator of the form $-\partial^2 + \underline{\underline{M}}^2$. Any such propagator, upon back-transformation to coordinate space, can only lead to a logarithmic behaviour for the vortex interactions at small and large distances, and consequently, cannot possibly reproduce the confining linear long-range intralayer interaction given in equation (8.42) of [4] and in [31]. The candidate [31] for a mass matrix $\underline{\varphi}^{\text{T}} \underline{\underline{m}}^2 \underline{\varphi} = J \sum_{i=1}^{N-1} (\varphi_i - \varphi_{i+1})^2$ has also been discussed in [1, 33, 34]. This candidate interaction is inspired by a discretization of the anisotropic 3D SG model [35], but it cannot reproduce the linear confining potential needed for the description of the Josephson-coupled case [1]. The layer-dependent transition temperature of this model is $T_{\text{c}} \propto N^{-1}$ and decreases with the number of layers, and for general N, the mass matrix $\underline{\underline{m}}^2$ also leads to different short- and long-range intralayer potentials as compared to equation (8) and cannot be used for the description of magnetically coupled N-layer systems, either [1]. Finally, let us note that a suitable model for the Josephson-coupled layered system could probably be constructed if the interlayer interaction term is represented by a compact variable, i.e. one coupling the phase (compact) fields between the 2D planes [17] and not the dual fields.

3. RG analysis of the multi-layer sine–Gordon model

The above statements on the MLSG model are based on the bare action where the coupling parameters of the theory are fixed. However, only a rigorous RG analysis enables one to construct the phase diagram in a reliable manner. For $N = 2$ layers, the phase structure and the vortex properties of the magnetically coupled layered system have already been considered with a real space RG approach [5] using a two-stage procedure, and a momentum space RG method [31] on the basis of the dilute gas approximation has also been used. Here, we apply a generalized multi-layer, multi-field Wegner–Houghton (WH) RG analysis developed by us for the layered SG type models [1, 33, 34, 36, 37] to the MLSG model with an arbitrary numbers of layers. In the construction of the WH RG equation, the blocking transformations [38] are realized by a successive elimination of the field fluctuations in the direction of decreasing momenta, in infinitesimal momentum shells, about the moving sharp momentum cut-off k (see [39]). The physical effects of the eliminated modes are transferred to the scale dependences of the coupling constants (e.g., $y \equiv y(k)$). The WH RG equation in the local potential approximation (LPA) for the MLSG model with N layers reads

$$(2 + k\partial_k)\tilde{V}_k = -\frac{1}{4\pi}\ln[\det(\delta_{ij} + \partial_{\varphi_i}\partial_{\varphi_j}\tilde{V}_k)], \qquad (9)$$

where we have defined the dimensionless blocked potential as $\tilde{V}_k \equiv k^{-2}V_k$. We make the following ansatz for the blocked potential:

$$\tilde{V}_k = \tfrac{1}{2}\tilde{G}_k\left(\sum_{n=1}^{N}\varphi_n\right)^2 + \tilde{U}_k(\varphi_1, \cdots \varphi_N), \qquad (10)$$

where the scale dependence is encoded in the dimensionless coupling constants $\tilde{y}(k)$ and $\tilde{G}(k)$ which are all related to their dimensionful (no tilde) counterparts by a relative factor k^{-2}. Inserting the ansatz (10) into equation (9), the right-hand side becomes periodic, while the left-hand side contains both periodic and non-periodic parts [34, 36].

In order to go beyond the dilute gas approximation, we calculate a mass-corrected UV approximation of equation (9) by expanding the logarithm of the determinant in the right-hand side of equation (9) in powers of the periodic part of the blocked potential. Because this procedure has been discussed at length in [33, 34, 36], we immediately state the result (cf equation (43) of [36]),

$$\tilde{y}(k) = \tilde{y}(\Lambda)\left(\frac{k^2 + NG}{\Lambda^2 + NG}\right)^{\frac{b^2}{N8\pi}}\left(\frac{k}{\Lambda}\right)^{\frac{(N-1)b^2}{N4\pi}-2}, \qquad (11)$$

with the initial value $\tilde{y}(\Lambda)$ at the UV cut-off $k = \Lambda$. Let us note that in our RG approach the dimensionful G and b^2 are scale-independent constants. We can immediately read off from equation (11) the critical value $b_c^2 = 8\pi/(1 - N^{-1})$ and the corresponding KTB temperature $T_{\text{KTB}}^{(N)} \sim b_c^{-2} = T_{\text{KTB}}^{\star}(1 - N^{-1})$. The fugacity $\tilde{y}$ is irrelevant (decreasing) for $b^2 > b_c^2$ and relevant (increasing) for $b^2 < b_c^2$ for decreasing scale k (see figure 1). Our RG approach provides a consistent scheme for calculating higher order corrections to the linearization in the periodic part of the blocked potential, which is equivalent to higher order corrections to the dilute gas approximation. For $N = 1$, the mass-corrected UV scaling law (11), obtained for the massive SG model, recovers the scaling obtained in [21, 40] (no phase transition).

4. Conclusion and summary

In conclusion, we propose the multi-layer sine–Gordon (MLSG) Lagrangian as a quantum field theoretical model for the vortex properties of magnetically coupled layered superconductors.
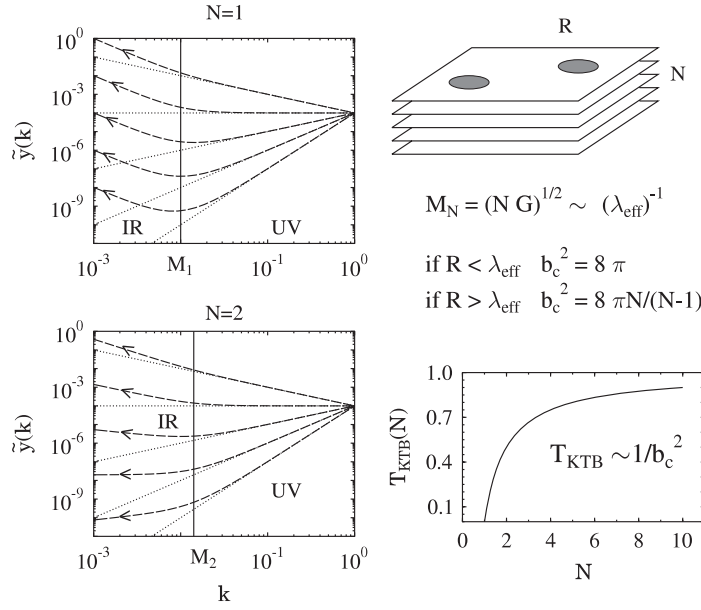
Figure 1. In the left panels, the mass-corrected scaling (see equation (11)) of the dimensionless Fourier amplitude $\tilde{y}$ of the MLSG model for $N = 1$ (top) and for $N = 2$ (bottom) layers is represented graphically for $b^2 = 4\pi, 8\pi, 12\pi, 16\pi, 20\pi$ (from top to bottom on each panel; see the dashed curves). We use $G = 0.0001$ in order to have the UV and IR regimes conveniently located on the plots, which start at the UV scale $\Lambda = 1$. The dotted line is the extrapolation of the UV ($k \gg M_N$) scaling to the IR ($k \ll M_N$) region. For $N = 1$ layers, $\tilde{y}$ is always relevant ($\sim k^{-2}$) in the IR. For $N = 2$ layers, $\tilde{y}$ is relevant for $b^2 < 16\pi$ in the IR and irrelevant for $b_c^2 > 16\pi$. Thus, the two-layer MLSG model undergoes a KTB type phase transition at $b_c^2 = 16\pi$. In general, the KTB transition temperature of the MLSG model is layer dependent, $T_{KTB}^{(N)} \sim (1 - N^{-1})$. If the system has a finite volume ($R < \infty$), the thermodynamic limit cannot be taken automatically and, as a simple realization of the finite size effect, a momentum scale $k_{min} \sim 1/R$ appears in the model. For $R < \lambda_{eff}$ (i.e. $k_{min} > \sqrt{NG} = M_N$), the phase structure of the MLSG model is determined by the UV scaling which predicts a KTB type phase transition at $b_c^2 = 8\pi$ for any number of layers.

Note that the MLSG model cannot be assumed to belong to the same universality class as the layered Ginzburg–Landau model [1], which entails a discretization of the Ginzburg–Landau model in one of the spatial directions. The mapping of the MLSG model onto the gas of topological defects is used to clarify the suitability of the MLSG model for magnetically coupled layered systems. We investigate the scaling laws for the MLSG model using a functional formulation of the Wegner–Houghton RG approach in the local potential approximation. The linearization of the RG flow in the periodic part of the blocked potential (and not in the full potential) enables us to incorporate the effect of the interlayer interaction into the mass-corrected UV scaling laws, which improve the dilute gas approximation. The mass-corrected Wegner–Houghton UV scaling laws indicate that for general interlayer interactions of the type of equations (3), one finds two phases separated by the critical value $b_c^2 = 8\pi/(1 - N^{-1})$, where N is the number of layers. This determines the layer dependence of the KTB transition temperature $T_{KTB}^{(N)} = T_{KTB}^{\star} (1 - N^{-1})$ in full agreement with [3, 5]. Perhaps further investigations of the MLSG model (e.g. beyond the local potential approximation) and other generalizations of the momentum space RG studies presented here could enrich our understanding of the layered structures.

Acknowledgments

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